Homework 13

1. Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

$$\int_0^{\infty} x^2 e^{-x}\, dx$$

2. Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

$$\int_0^{\infty} \cos \pi x\, dx$$

3. Determine whether the following integral converges or diverges.(Using the limit comparison test to solve the problem.)

$$\int_1^{\infty} \frac{x^2 + 3x}{\sqrt{x^5 + 1}}\, dx$$

Sol：

1.

$$\int_0^{\infty} x^2 e^{-x}\,dx$$

$$= \lim_{b\to\infty} \int_0^{b} x^2 e^{-x}\,dx$$

$$= \lim_{b\to\infty} [-e^{-x}(x^2 + 2x + 2)]_0^b$$

$$= \lim_{b\to\infty} \left(-\frac{b^2 + 2b + 2}{e^b} + 2\right)$$

$$= 2$$

2.

$$\int_0^{\infty} \cos \pi x\,dx$$

$$= \lim_{b\to\infty} [\frac{1}{\pi} \sin \pi x]_0^b$$

→ Diverges

3.

$$\lim_{n\to\infty} \frac{(\frac{x^2 + 3x}{\sqrt{x^5 + 1}})}{(\frac{1}{x^{1/2}})} = 1$$

$\int_1^{\infty} \frac{1}{x^{1/2}}\,dx$ is divergent, then by the limit comparison test so does $\int_1^{\infty} \frac{x^2+3x}{\sqrt{x^5+1}}\,dx$.